Bargaining While Learning About New Arrivals

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Abstract

We study dynamic bargaining with uncertainty over the buyer's valuation and the seller's outside option. A long-lived seller makes offers to a long-lived buyer whose value is private information. There may exist a short-lived buyer whose value is higher than that of the long-lived buyer. The arrival of the short-lived buyer, if she exists, is determined by a Poisson process. We characterize the unique equilibrium. The equilibrium displays interesting price fluctuations: in some periods, the seller charges a high price unacceptable to the long-lived buyer, in the hope that the short-lived buyer will appear in that period; in the other periods, he offers a price attractive to some values of the long-lived buyer. The price dynamics result from the interaction between two learning processes: exogenous learning about the existence of short-lived buyers, and endogenous learning about the long-lived buyer's value.

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1 Introduction

The arrival of new buyers has a great impact on the market where payoffs are determined through bargaining. The seller may have an incentive to delay the trade and wait for other buyers to arrive, whereas the present buyer will lose bargaining power due to the competition that arises from the arrival of new buyers. Further, in many situations, the likelihood of new arrivals may be uncertain.

Consider the following story, for instance. Suppose a seller and a buyer are negotiating on the price of a house. There may be another buyer entering the market at some point. This new buyer, if he exists, places a very high value on the house and has an urgent demand.[1] However, it is not clear whether such a new buyer exists. While the likelihood of new arrivals is initially uncertain, it could be learned over time. When the seller is very optimistic about the existence of new arrivals, she wants to wait for such a new buyer by charging a high price that the current buyer would never take. As time elapses and no new buyer shows up, the seller becomes pessimistic, and so she begins to treat the current buyer seriously. Then in this environment, how do the seller's exogenous learning about new arrivals and her endogenous learning about the current buyer's value affect the transaction time and the equilibrium pricing path?

In this paper, we study a bargaining model, highlighting the interaction between the seller's exogenous learning about the existence of new arrivals and her endogenous learning about the current buyer's value. A long-lived seller, possessing a single unit of an indivisible durable good, makes a price offer in each period. There is a long-lived buyer whose reservation value is his private information. Observing the seller's offer, the long-lived buyer decides whether to accept it. In addition, a short-lived buyer with a high reserve price may exist. If such a short-lived buyer exists, his arrival is governed by a Poisson process. If the current offer is rejected by the long-lived buyer, the arriving short-lived buyer makes the purchase decision immediately and then leaves the market. The game ends once the good is sold.

We show that the model has a unique perfect Bayesian equilibrium and in the equilibrium, the standard Coase conjecture fails when learning about new arrivals is non-trivial. [2] That is, when the seller can make offers arbitrarily frequently, the initial price is bounded away from the seller' reservation value. This is because waiting for new arrivals serves as a non-trivial outside option of the seller. As long as the likelihood of arrivals is non-trivial, terminating the bargaining by offering a very low price is suboptimal to him. Since the learning speed of the existence of new arrivals is exogenous, the value of the outside option smoothly declines over time. Therefore, the seller slowly screens the long-lived buyer, which results in a strategic delay of the transaction.

We also characterize the equilibrium price dynamics. We show that the interaction between the seller's exogenous learning about the existence of new arrivals and her endogenous learning about the current buyer's value determines the equilibrium price dynamics. When the seller is optimistic about the new arrivals, she charges a price equal to the short-lived buyer's value. Such a high price offer is effectively made to the potential short-lived buyer

[1] Foreign investors from Asia are examples of new buyers, who are credited for driving the real estate market in Southern California recently. They often accept high all-cash offers, suggesting that they have high values and urgent demands.

[2] See Coase (1972), Bulow (1982), Gul, Sonnenschein and Wilson (1986) and Ausubel and Deneckere (1989).

only. We call such an offer a *waiting offer*. Charging a waiting offer forever, however, is suboptimal. If the good is not sold by charging a waiting offer for a very long time, the posterior belief about the existence of new arrivals drops below some threshold point. Then the seller finds that is is more valuable to screen the long-lived buyer by cutting the price. We call such an offer a *screening offer*. Of course, the screening offer may be accepted by an arriving short-lived buyer, if it is rejected by the long-lived buyer. So if the good is still not sold at a screening offer, both the seller and the long-lived buyer will adjust downward their beliefs about the existence of the new arrivals. Apparently, when the likelihood of new arrivals is high enough, the seller chooses to post a waiting offer. When the likelihood of new arrivals is relatively low, the seller chooses to post a screening offer.

Specifically, Proposition 6 shows that if the time interval between two offers is sufficiently small, screening offers cannot be charged in more than two consecutive periods. The logic is similar to the Coase conjecture: when the seller can make offers arbitrarily frequently, he always prefers to speed up the screening of the long-lived buyer. As a result, after being rejected once or twice, the seller believes that the long-lived buyer's value is too low so that he switches to wait for new arrivals.

Dynamic bargaining models allowing new arrivals have not drawn economists' attentions until recently. Sobel (1990) considers a dynamic pricing model in which buyers keep arriving over time. In an equilibrium, the seller serves high value buyers most of the time and periodically serves low value buyers, so the equilibrium price fluctuates over time. However, in Sobel (1990), the price fluctuation is driven by the accumulation of arrivals instead of learning. Inderst (2008) assumes that the seller can choose whether to keep the original buyer or to switch to the new long-lived buyer who arrives with a known probability. If he switches, he starts a new screening, and thus the Coase conjecture is robust in this perturbation. Fuchs and Skrzypacz (2010) also consider a bargaining game between a seller and a buyer for a trade of one unit of an indivisible good, in which a stochastic event arrives in each period with a known probability. If no event arrives, the seller posts a price to the buyer whose value is his private information. Conditional on arrival, the seller's expected value depends on his belief about the buyer's value. One example of the stochastic event is the arrival of a new buyer whose value is unknown. Conditional on arrival, two buyers bid for the good in an auction, and the seller's expected revenue from the auction depends on the current buyer's value. They show that there is a delay in equilibrium and the seller slowly screens out buyers with higher valuations. They characterize the continuous time limit of the price path, which turns out to be very tractable. The existence of the arrival is commonly known in these models, so equilibrium prices are strictly decreasing. As we show in our model, learning about the existence of the short-lived buyer leads to price fluctuations in equilibrium.

In a parallel study by Faingold, Liu and Shi (2011), the buyer has an outside option whose existence is learned over time. The belief about the existence of the buyer's outside option decreases over time, and the seller smoothly changes the price. Our paper is different from Faingold, Liu and Shi (2011) in that the seller posts a price at the very beginning of each period, and conditional on arrival, the seller has to commit to the price. We show that the combination of learning and this marginal commitment power generates price fluctuations in our model.

In addition, a number of papers have studied the problem of a seller who learns about the demand it faces.[3] Mason and Valimaki (2011) studied a monopolist who faces a sequence of short-lived buyers with an unknown arrival rate and a commonly known value distribution. The seller posts a price in each period and learns the buyers' arrival rate, which depends on both an unknown parameter of the Poisson process and the posted price. As time passes without a sale, the seller becomes more pessimistic about the arrival rate and therefore smoothly lowers the posted price. In this paper, the seller faces not only a sequence of short-lived buyers with degenerated value but an unknown arrival rate but also a long-lived buyer with privately known value. Hence, we see Mason and Valimaki (2011) as a complement to our research, rather than a substitute.

The rest of this paper is organized as follows. In sections 2 and 3, we introduce the bargaining model with learning about new arrivals. Section 4 is devoted to the no learning case as a benchmark. In section 5, we construct the unique equilibrium of the bargaining game with learning and show the price fluctuation as an inevitable phenomenon. Section 6 concludes. All omitted proofs are presented in the appendix.

2 The Model

Time is discrete and is indexed by $t \in \{0, 1, 2, \dots\}$. Any period has the same length Δ. A long-lived seller has one unit of a durable good for sale. The value of the good to the seller is 0. There is a long-lived buyer, and the value of the good to this buyer, v, is a random variable with support $[\underline{v}, 1]$. In this paper, we assume $\underline{v} > 0$, so we focus on the "gap case" bargaining model. The value v is drawn by Nature at the beginning of the game according to a commonly known distribution, and the long-lived buyer privately knows it. Let $F(v)$ and $f(v)$ be the cumulative distribution function and the density function of v, respectively. Assume that $f(v) \in (0, \bar{f}]$ for all $v \in [\underline{v}, 1)$. In addition, we assume the density function satisfies $f(1) = f'(1) = 0$.[4] There may exist a short-lived buyer whose value of the good is 1. The long-lived seller and the long-lived buyer share a common prior belief about the existence of the short-lived buyer, denoted by $\alpha_0 \in (0, 1]$.[5] Conditional on his existence, the arrival of the short-lived buyer is determined by a Poisson process. Specifically, in each period, the short-lived buyer enters the market with probability $\lambda\Delta$ if he exists. Intuitively, when the initial likelihood of arrival is small enough, the seller may ignore the possibility of arrival and focus on screening the long-lived buyer. To avoid a trivial case, we make the following assumption.

Assumption 1 (non-trivial learning)**.** $\underline{v} < \frac{\lambda\alpha_0}{\lambda\alpha_0 + r}$.

In each period t, the seller first announces a price p_t. The long-lived buyer, observing p_t, decides whether to accept or reject the offer. If he rejects it, the short-lived buyer may arrive (with probability $\lambda\Delta$) if he exists. Conditional on his arrival, the short-lived buyer decides whether to accept the current price if the good is available. If there is no transaction in period t, the game enters period $t + 1$. Once the good is sold, the game ends.

The long-lived seller and the long-lived buyer share the same discount factor $e^{-r\Delta}$, where $r > 0$. If the transaction happens with price p_t in period t, the seller's payoff is

[3]See Rothschild (1974), McLennan (1984), Bergemann and Valimaki (1994, 2006), among others.

[4]This assumption helps us to show the existence and uniqueness of the equilibrium, so that we can focus on the discussion of equilibrium properties.

[5]When $\alpha_0 = 0$, the game is the canonical Coasian bargaining model.

$e^{-tr\Delta}p_t$. If the game ends with the long-lived buyer buying the good at price p_t, then the long-lived buyer's payoff is $e^{-tr\Delta}(v - p_t)$. If the game ends with the short-lived buyer accepting an offer, the long-lived buyer's payoff is zero. The short-lived buyer's payoff is

$$v_s = \begin{cases} 1 - p_t, & \text{if the good is available and he accepts it,} \\ 0, & \text{otherwise.} \end{cases}$$

Define $\mathcal{H}^t$ as the set of period t histories such that no transaction happens, so $h^t \in \mathcal{H}^t$ is a sequence of price $\{p_\tau\}_{\tau=0}^{t-1}$, which has not been accepted by either the long-lived or the short-lived buyer. Let $\mathcal{H} \equiv \cup_t \mathcal{H}^t$. A strategy of the seller is a map from the histories of rejected prices to price offers in the current period. It is obvious that any price strictly greater than 1 is dominated by the price 1 and thus is suboptimal, because neither the long-lived buyer nor the potential short-lived buyer will accept a price strictly greater than 1. Hence, we restrict the price space on $[0, 1]$. Denote a pure strategy of the seller by the mapping $P : \mathcal{H} \to [0, 1]$. A behavior strategy of the long-lived buyer specifies whether to accept the offer in any period t, given the price p_t, past rejected prices $\{p_\tau\}_{\tau=0}^{t-1}$ and his value v. Formally, let $A : \mathcal{H} \times [0, 1] \times [\underline{v}, 1] \to \{0, 1\}$, where $A(h^t, p, v) = 0$ means that the long-lived buyer with value v and history h^t rejects the offer p_t at period t. The short-lived buyer accepts any offer $p_t \leq 1$ if it is available.

3 Equilibrium

3.1 Weak Markov Equilibrium

A perfect Bayesian equilibrium (henceforth "PBE") in this game consists of a strategy profile, a system of beliefs about the long-lived buyer's value, and a system of beliefs about the existence of the short-lived buyer. In a PBE, given the systems of beliefs, all players behave sequentially rationally in any information set; and given the strategy profile, the system of beliefs is calculated by Bayes' rule, whenever it can be applied.

Because whether the short-lived buyer shows up is publicly observable to both the long-lived seller and the long-lived buyer, they share the same belief about the existence of the short-lived buyer after any history h^t. Denote such a belief at the beginning of period t by α_t, then if no short-lived buyer arrives and no transaction happens in period t, both the long-lived seller and the long-lived buyer update their beliefs by Bayes' rule as

$$\alpha_{t+1} = \frac{\alpha_t(1 - \lambda\Delta)}{1 - \alpha_t\lambda\Delta}. \tag{1}$$

In the rest of the paper, we sometimes use α and α' to denote the belief about the existence of the short-lived buyer in the current period and in the next period, respectively. Similarly, primes are used to denote next-period values of other variables. Fixing a $\Delta > 0$, the updating rule of α implies that there are countably many realizations of α. We denote by $B_\Delta(\hat{\alpha})$ the set of realizations of α, which can be generated from $\hat{\alpha}$ by the Bayes' rule given Δ.

The belief α_{t+1} summarizes the information about the short-lived buyer's existence from the no transaction history at the end of period t. The following Lemma, extending the insight of Fudenberg, Levine, and Tirole (1985) to this model, claims that the long-lived seller's belief about the long-lived buyer's value after any no transaction history is a truncated sample of the original distribution. As a consequence, the long-lived seller's posterior belief about the long-lived buyer's value is summarized by the upper bound of the truncated distribution.

Lemma 1 (Conditional Skimming Property)**.** *In any perfect Bayesian equilibrium, conditional on the current α, if the long-lived buyer with value v accepts an offer, any long-lived buyer with $v' > v$ must accept it.*

The conditional skimming property implies that in any perfect Bayesian equilibrium, after any relevant history of offered prices, there exists a value k such that the long-lived buyer rejects all of these offers if and only if his value $v \leq k$. The intuition is as follows. The long-lived buyer's benefit of waiting comes from the decline in the future price, which does not depend on his value. But the cost of waiting results from postponing consumption, which is increasing in the long-lived buyer's value v. Put differently, it is more costly for the high value long-lived buyer to delay his consumption than it is for the low value buyers.

The conditional skimming property implies that in a PBE, the long-lived buyer's action in any period t depends only on his value, the current price, and the current belief about the existence of short-lived buyers. In particular, after any history h^t in a PBE, if the long-lived buyer with value k is indifferent between taking the current offer and waiting for future offers, the long-lived buyer with a value larger than k strictly prefers the current offer. Hence, the cutoff k summarizes the payoff relevant information about the long-lived buyer's value, and it is common knowledge that the seller's belief about the long-lived buyer's value is distributed according to a truncated distribution of $F(v)$ with support $[\underline{v}, k]$. As a result, it is natural to consider that in a PBE, players will condition their actions in any information set only on two state variables (k, α), the highest value at which the long-lived buyer has not bought and the belief about the existence of the short-lived buyer. In the literature, a PBE with such conditions is called a strong Markov equilibrium. However, as shown in Fudenberg, Levine, and Tirole (1985), a strong Markov equilibrium does not generally exist. Therefore, in this paper, we use the weak-Markov equilibrium as the solution concept of the model.

Definition 1. *A strategy profile (P, A) is a weak Markov equilibrium (henceforth equilibrium), if it is a PBE and there exist two functions*

$$\sigma : [\underline{v}, 1] \times B_\Delta(\alpha_0) \times [0, 1] \to [0, 1] \quad \textit{and} \quad \kappa : [0, 1] \times B_\Delta(\alpha_0) \to [\underline{v}, 1],$$

such that, in any period t,

1. *$P(h^t) = \sigma(k_t, \alpha_t, p_{t-1})$, if $h^t \in \mathcal{H}^t$ induces (k_t, p_{t-1}); and*
2. *for any $h^t \in \mathcal{H}^t$ and any price p, $A(h^t, p, v) = 0$ if and only if $v \leq \kappa(p, \alpha_t)$.*

We say (σ, κ) describes an equilibrium.

The functions σ and κ describe the seller's equilibrium pricing strategy and the long-lived buyer's equilibrium acceptance rule, respectively. The first requirement of the equilibrium definition states that the seller's price in period t depends only on her belief about the long-lived buyer's value, her belief about the existence of the short-lived buyer, and the last period rejected price p_{t-1}. In addition, we will show that on the equilibrium path, the continuation play depends only on (k_t, α_t), so we abuse notation by treating σ as a function of (k_t, α_t) but not p_{t-1} on the equilibrium path. However, we keep in mind that on the off-equilibrium path, σ may depend on p_{t-1}. The second requirement of the equilibrium definition shows that the long-lived buyer will employ a cutoff rule, on and off the equilibrium path. Given any price and the belief about the existence of the short-lived buyer in period t, the long-lived buyer at value $\kappa(p, \alpha_t)$ will be indifferent between taking the offer p and waiting.

3.2 Screening Offer and Waiting Offer

Before moving to the formal analysis of the model, in this section, we introduce some preliminary analysis, as well as some notations. Let's first consider the belief about the existence of the short-lived buyer. Conditional on his existence, the short-lived buyer arrives in any period with probability $\lambda\Delta$. Therefore, no transaction in period t will make both the seller and the long-lived buyer shift downward their belief about the existence of the short-lived buyer according to (1).

Second, in the standard bargaining models without the short-lived buyer, the equilibrium price sequence is strictly decreasing over time because of the skimming property. Consequently, the upper limit of the support of the seller's belief about the long-lived buyer's value is also strictly decreasing over time. In our model, however, the seller may charge a high price in equilibrium, which the long-lived buyer rejects for sure, because the potential short-lived buyer places a higher value on the good and thus takes the high price offer. Hence, the upper limit of the support of the seller's belief about the long-lived buyer's value may not be strictly decreasing. If the seller decides to charge such a price, he will set a price at 1, the short-lived buyer's value, and the long-lived buyer rejects the offer for sure.

Definition 2. *A price is a screening offer if and only if the long-lived buyer with some value $v \in [\underline{v}, k]$ may accept it, given α. The price $p = 1$ is a waiting offer.*

If the seller provides a waiting offer, the long-lived buyer will reject it in an equilibrium, no matter what her value is. Therefore, after a waiting offer, the seller's belief about the long-lived buyer's value does not change. If the seller provides a screening offer, there is a positive measure of values with which the long-lived buyer will accept it. As a result, no transaction causes the seller to update his belief about the long-lived buyer's value as

$$k' = \begin{cases} \kappa(p, \alpha), & \text{if } p \text{ is a screening offer,} \\ k, & \text{if } p = 1 \text{ is the waiting offer.} \end{cases} \tag{2}$$

For a given state variable vector (k, α), denote the seller's value from optimally charging a screening offer by $V(k, \alpha)$. If the seller chooses a waiting offer, we denote his value by $J(k, \alpha)$. Therefore, given (k, α), the seller charges the optimal screening offer if and only if $V(k, \alpha) \geq J(k, \alpha)$. Specifically, given the buyer's cutoff strategy $\kappa(p, \alpha)$, the seller's problem is

$$R(k, \alpha) = \max\{J(k, \alpha), V(k, \alpha)\}, \tag{3}$$

where

$$J(k, \alpha) = \lambda\alpha\Delta + (1 - \lambda\alpha\Delta)e^{-r\Delta}R(k', \alpha') \tag{4}$$

and

$$\begin{aligned} & V(k,\alpha) \\ = \; & \max_{p}\{[\frac{F(k)-F(\kappa(p,\alpha))}{F(k)}+\frac{F(\kappa(p,\alpha))}{F(k)}\alpha\lambda\Delta]p \\ & \qquad +\frac{F(\kappa(p,\alpha))}{F(k)}(1-\alpha\lambda\Delta)e^{-r\Delta}R(\kappa(p,\alpha),\alpha')\}. \end{aligned} \tag{5}$$

Denote the optimal screening offer given (k,α) by $\sigma^s(k,\alpha)$ (that is, $\sigma^s(k,\alpha)$ is the optimal policy for equation (5)), then

$$\sigma(k,\alpha)=\begin{cases}\sigma^s(k,\alpha), & \text{if} \quad V(k,\alpha)\geq J(k,\alpha),\\ 1, & \text{if} \quad V(k,\alpha)<J(k,\alpha).\end{cases}$$

Using the highest price accepted by the long-lived buyer with value v given α, we reformulate (5) as

$$\begin{aligned} & V(k,\alpha) \\ = \; & \max_{\underline{v}\leq k'\leq k}\{[\frac{F(k)-F(k')}{F(k)}+\frac{F(k')}{F(k)}\alpha\lambda\Delta]\kappa^{-1}(k'|\alpha) \\ & \qquad +\frac{F(k')}{F(k)}(1-\alpha\lambda\Delta)e^{-r\Delta}R(k',\alpha')\}. \end{aligned} \tag{6}$$

Hence, the policy correspondence of the seller's problem is

$$T(k,\alpha)=\begin{cases}k, & \text{if} \quad \sigma(k,\alpha)=1,\\ T_s(k,\alpha), & \text{otherwise},\end{cases}$$

where $T_s(k,\alpha)$ is the maximum of the set of solutions to problem (6).

By definition, when the seller charges a waiting offer, the long-lived buyer will reject it for sure, no matter what her value is. When the seller posts a screening offer, the best response of the long-lived buyer with value v can be characterized by the following indifferent condition:

$$v-\sigma(k,\alpha)=e^{-r\Delta}(1-\alpha\lambda\Delta)U(v,k',\alpha'|\sigma),$$

where

$$U(v,k,\alpha|\sigma)=\max\{v-\sigma(k,\alpha),e^{-r\Delta}(1-\alpha\lambda\Delta)U(v,k',\alpha'|\sigma)\}$$

denotes the continuation value of the long-lived buyer with value v, when the state variable vector is (k,α) and the seller follows pricing strategy P. As argued above, the equilibrium prices may not be strictly decreasing over time, because the seller may charge waiting offers. Thus, given a price sequence, the indifferent condition of the long-lived buyer can be written as

$$v-p_t=e^{-(n+1)r\Delta}\left[\prod_{i=t}^{t+n}(1-\alpha_i\lambda\Delta)\right](v-p_{t+n+1}),$$

where on the right-hand side, the long-lived buyer waits for $n+1$ periods and takes the risk that the short-lived buyer arrives between period t and period $t+n$.

4 Bargaining Without Learning

In this section, we assume $\alpha_0 = 1$. That is, it is common knowledge that the short-lived buyer exists. Though the arrival time of the short-lived buyer is still random, there is no learning about the existence of the short-lived buyer. Therefore, the analysis in this section provides a good benchmark for us to show the effect of learning about the existence of the short-lived player in Section 5.

Because the seller is sure that the short-lived buyer exists, he can charge the waiting offer forever and expect a time-invariant value. This is a non-trivial outside option for him. Denote such an outside option by J_0, then

$$J_0 = \lambda\Delta + (1 - \lambda\Delta)e^{-r\Delta} J_0.$$

When Δ is small, $e^{-r\Delta}$ can be approximated by $1 - r\Delta$, so the value of waiting for arrivals J_0 can be approximated by $\frac{\lambda}{r+\lambda}$ for small $\Delta > 0$. Furthermore, by Assumption 1, we have $J_0 > \underline{v}$. That is to say, the seller prefers to wait for arrivals rather than trading at an extremely low price (close to $\underline{v}$) immediately. Consequently, once the seller believes the long-lived buyer's value is sufficiently low, he prefers to stop screening him and wait for arrivals by using waiting offers.

Once the seller charges a waiting offer in period t, his belief about the long-lived buyer's value does not change. Hence, if there is no transaction in period t, his belief about the long-lived buyer's value in period $t + 1$ is the same as that in period t. Therefore, if it is optimal for the seller to charge the waiting offer in period t, it is optimal for him to charge the waiting offer in period $\tau > t$. Consequently, the seller's problem of when to charge the waiting offer is a stopping time problem. Furthermore, $\frac{\lambda}{r+\lambda} < 1$, so the seller will charge screening offers before switching to the waiting offer forever. Hence, before switching to the waiting offer, the seller's problem is almost identical to that in the canonical Coase bargaining problem. The following Proposition summarizes this intuition.

Proposition 1. *When $\alpha_0 = 1$, an equilibrium exists. Generically, the equilibrium is the unique PBE of the model. Furthermore, there exists an integer N such that, in the equilibrium,*

1. *the seller posts screening offers in the first N periods,*
2. *the price decreases in the first N periods, and*
3. *from the $N + 1$ period on, the seller switches to the waiting offer forever.*

Proposition 1 implies that, in equilibrium, there are two phases: the screening phase and the waiting phase. That is to say, the seller first screens the long-lived buyer for finitely many periods by gradually cutting the price. Once the price reaches a certain cutoff level, the seller believes that the value of the long-lived buyer is sufficiently low so that he will give up the long-lived buyer and wait for the arrivals by charging a high price in the future. In the following proposition, we show that the screening process ends very quickly when the time interval of two consecutive periods converges to zero and the initial price converges to $\frac{\lambda}{\lambda+r}$. Hence, a modified Coase conjecture holds, and there is no strategic delay in the equilibrium.

Proposition 2. *In the equilibrium, the following properties hold:*

1. $p_t \geq \frac{\lambda}{\lambda+r}$ *for any* $\Delta > 0, t \geq 0$.

2. *(No Strategic Delay)* p_0 *goes to* $\frac{\lambda}{\lambda+r}$ *as* Δ *goes to zero.*

5 Bargaining While Learning About Arrivals

In this section, we study the bargaining game when $\alpha_0 \in (0,1)$. In this case, the interaction between the seller's exogenous learning about the existence of the short-lived buyer and his endogenous learning about the long-lived buyer's value has significant effects on the equilibrium characterization, the equilibrium bargaining outcome, and the equilibrium pricing path.

5.1 Equilibrium Characterization

Different from the no learning case, posting the waiting offer forever from some period on cannot be part of an equilibrium. The reason is as follows. By posting a waiting offer, the seller learns the likelihood of arrival only but not the value of the long-lived buyer. After finitely many periods, the seller believes that the likelihood of arrival is negligible, and his expected payoff from waiting longer for a new arrival is almost zero! As a result, the seller can simply charge a price $p = \underline{v}$ to trade with the long-lived buyer immediately. Hence, the "waiting offers forever" strategy cannot be part of an equilibrium in any continuation game. In addition, the seller's equilibrium payoff in any continuation game with α is bounded below by $\max\{J_0(\alpha), \underline{v}\}$, so for sufficiently large $\lambda\alpha$, the equilibrium screening price is bounded away from $\underline{v}$.

Define

$$\alpha^{\dagger} \equiv \max\{\alpha | \alpha \in B_{\Delta}(\alpha_0) \text{ and } \underline{v} \geq \alpha\lambda\Delta + (1-\alpha\lambda\Delta)e^{-r\Delta}\underline{v}\}$$

to be the largest realization of α, at which it is better for the seller to set a price at $\underline{v}$ (to finish the bargaining) than to charge the waiting offer once and then set a price at $\underline{v}$.

Lemma 2. *For any* $k \in [\underline{v}, 1]$*, if* $\alpha \in B_{\Delta}(\alpha^{\dagger}) = B_{\Delta}(\alpha_0) \cap \{\alpha | \alpha \leq \alpha^{\dagger}\}$*, then the seller does not charge the waiting offer in equilibrium.*

Lemma 2 implies that when α is small enough, the seller charges only screening offers in any equilibrium. Therefore, the waiting offer, if charged in an equilibrium, is a temporary phenomenon. Because when $\alpha > \alpha^{\dagger}$, the seller will not charge a price $p = \underline{v}$ to finish the bargaining and, there is a positive probability that some $\alpha \leq \alpha^{\dagger}$ is reached on the equilibrium path. Therefore, to analyze players' equilibrium behaviors when α is large, we first need to characterize the equilibrium behavior when $\alpha \leq \alpha^{\dagger}$. The following Proposition provides a detailed characterization of the continuation equilibrium for such small α's.

Proposition 3. *In any continuation game starting at* $(k, \alpha) \in [\underline{v}, 1] \times B_{\Delta}\left(\alpha^{\dagger}\right)$*, an equilibrium exists, and it satisfies the following properties:*

1. *It is the unique PBE of the continuation game.*

2. *The game ends in finitely many periods.*

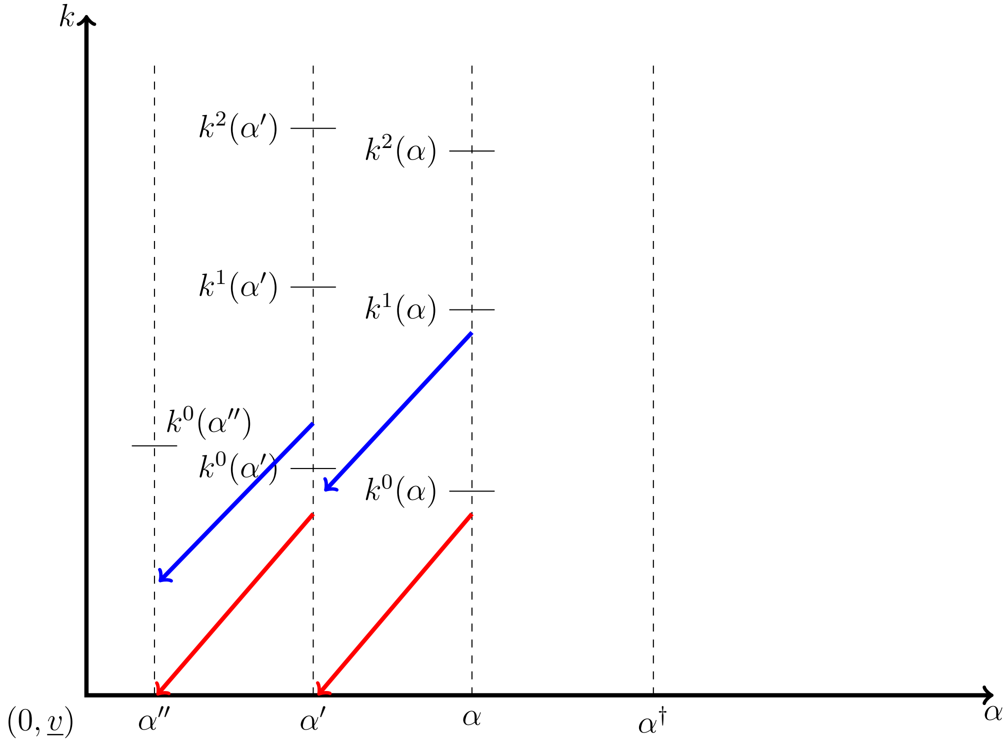

Figure 1: equilibrium construction in the learning case

When $\alpha \leq \alpha^\dagger$, the belief about the existence of the short-lived buyer is so low that the long-lived seller will charge only screening offers. We can construct the unique equilibrium in the same way as in the no learning case. First, for each $\alpha \in B_\Delta(\alpha^\dagger)$, we define $k^0(\alpha)$ as the maximum k such that a seller at (k, α) optimally charges $\underline{v}$ immediately rather than making other screening and waiting offers, and therefore, the game ends in one period. The reason is that the seller is so pessimistic about both the long-lived buyer's value and the short-lived buyer's arrival that he prefers to obtain $\underline{v}$ immediately. As a result, the game ends within one period. Similarly, fixing any $\alpha \in B_\Delta(\alpha^\dagger)$, we define $k^1(\alpha)$, such that when $k \in (k^0(\alpha], k^1(\alpha))$, the seller's optimal screening offer is a price $\sigma(k, \alpha)$, and there is a marginal type $k' \in [\underline{v}, k^0(\alpha'))$ who is indifferent between taking $\sigma(k, \alpha)$ and waiting for one more period. We show there is an integer N, such that inductively applying this construction method for N times, we have $k^N(\alpha) > 1$ for any $\alpha \in B_\Delta(\alpha^\dagger)$. This idea of equilibrium construction is illustrated in Figure 1.

In the original game, it takes finitely many periods for the belief to decrease from $\alpha_0 < 1$ to $\alpha^\dagger$ regardless of the players' strategy profile. Hence, Corollary 1 immediately results from Proposition 3.

Corollary 1. *In any perfect Bayesian equilibrium, the game ends in finitely many periods.*

The next Proposition extends the equilibrium construction to the space of (k, α), where $(k, \alpha) \in [\underline{v}, 1] \times B_\Delta(\alpha_0) \setminus B_\Delta(\alpha^\dagger)$, and shows that it is essentially the unique PBE.

Proposition 4. *The equilibrium exists. Generically, it is the unique PBE.*

The equilibrium construction and the proof of its uniqueness are presented in the Appendix. Here we present only the idea of construction as follows. First, since the unique equilibrium strategy profile, (σ, κ), is constructed for any continuation game starting at $(k, \alpha^{\dagger})$ $(k \in [\underline{v}, 1])$, the equilibrium payoff of the long-lived buyer with any value is specified under (σ, κ). Second, extend the long-lived buyer's strategy profile $\kappa(p, \alpha)$ to $\alpha^2 = \min\{\alpha \in B_{\Delta}(\alpha_0), \alpha > \alpha^{\dagger}\}$ such that $\kappa(p, \alpha^2)$ is indifferent between taking p in this period and obtaining his continuation payoff, $U\left(\kappa(p, \alpha^2), \kappa(p, \alpha^2), \alpha^{\dagger} | \sigma, \kappa\right)$, in the next period. Third, extend the seller's screening strategy $\sigma^s(\cdot, \alpha)$ to α^2. Fourth, for any $k \in [\underline{v}, 1]$, compare the seller's values $V(k, \alpha^2)$ induced by the optimal screening offer and $J(k, \alpha^2)$ induced by a waiting offer, and define $R(k, \alpha^2) = V(k, \alpha^2), \sigma(k, \alpha^2) = \sigma^s(k, \alpha^2)$ if $V(k, \alpha^2) \geq J(k, \alpha^2)$, and $R(k, \alpha^2) = J(k, \alpha^2), \sigma(k, \alpha^2) = 1$ otherwise. Finally, compute the buyer's payoff, $U(k, k, \alpha^2 | \sigma, \kappa)$ for each $k \in [\underline{v}, 1]$. Since it takes finitely many steps to go from α_0 to $\alpha^{\dagger}$, we can repeat the above construction method for finitely many times and extend (σ, κ) to α_0. We can essentially find a unique path of $\{k_t, \alpha_t\}_{t=0}^{N^*}$ from $(1, \alpha_0)$ to $(\underline{v}, \alpha_{\tau})$ where $\tau \leq N^*$.

5.2 Strategic Delay and Price Fluctuation

In this subsection, we analyze properties of the equilibrium and demonstrate the role of the interaction between the seller's exogenous learning about the existence of the short-lived buyer and his endogenous learning about the long-lived buyer's value. When the likelihood of new arrivals is sufficiently high, waiting for arrivals is a non-trivial outside option, so that the seller has no incentive to post a sufficiently low price to ensure a trade with the long-lived buyer takes place immediately, even though the time interval between the two offers is arbitrarily small. The following Proposition formalizes this intuition. Therefore, with the potential new arrivals, there are strategic delays of trades.

Proposition 5 (Strategic Delay with Non-trivial Learning)**.** *For any Δ and $T > 0$, there is an $\bar{\alpha} < 1$ such that, for any $\alpha_0 \in (\bar{\alpha}, 1)$, the equilibrium price $P_t > \underline{v}$ for any $t < T$.*

From Lemma 2, we know that, after finitely many periods, the seller charges only screening offers in the equilibrium. Hence, in equilibrium, there are two phases. In the second phase, the seller believes that the likelihood of arrivals is small and uses screening offer only. In the first phase, the seller may use both waiting offers and screening offers. Since the waiting price is high, the equilibrium price dynamics in the first phase may exhibit jumps. Intuitively, when the likelihood of new arrivals measured by α is large, the waiting offer may be optimal, because the expected revenue from selling the good to the short-lived buyer is high. When charging the waiting offer, only the belief about the arrivals changes, and the seller's belief about the long-lived buyer's value does not change. Hence, after charging waiting offers for a while, α goes down over time, the waiting offer becomes less attractive, and the seller will switch to screening offers. However, after several periods with screening offers, α may become relatively large (compared with k), so the waiting offer may become the optimal choice for the seller again. Therefore, the equilibrium price may exhibit fluctuation: some decreasing screening prices are followed by the waiting offer in a number of periods, and then even lower screening prices are charged. Unfortunately, it is well known that fully analyzing the equilibrium strategies in a dynamic pricing game is in general impossible. Hence, we focus on the games that satisfy certain conditions. First, following Fuchs and Skrzypacz (2010), we focus on the games with atomless limits.

Condition 1 (Atomless Limit)**.** *In the equilibrium, for all $t > 0$, if the seller uses screening offers in period t, $t+1$ and $t+2$, then $\kappa_{t+1} - \kappa_{t+2}$ is $O(\Delta)$.*

Condition 1 simply means that, when the seller consecutively uses screening offers, the possibility of trade in each period is not very large. That is to say, the seller will smoothly screen the long-lived buyer. Second, we impose a smoothness condition.

Condition 2 (Smoothness)**.** *For all $t > 0$, $\kappa(p, \alpha) = \kappa(p', \alpha')$, we have $p - p'$ is $O(\Delta)$.*

Condition 2 means that when the likelihood of arrivals changes slightly, the long-lived buyer's equilibrium strategy does not dramatically change. Under both conditions, we can further characterize the equilibrium price dynamics in the first phase: when the time interval between two consecutive periods is small, in the first phase of the equilibrium, the seller posts screening offers for one or two periods, then switches to waiting offers for some periods, and then switches back to screening offers for one or two periods, and then switch back to waiting offers. This frequent switching between waiting offers and screening offers stops at the end of the first phase. The following Proposition formally shows this pricing pattern in the first phase.

Proposition 6 (Price Fluctuation)**.** *Suppose Δ is small and $\alpha_0 \in (\alpha^{\dagger}, 1)$. Suppose also both condition 1 and condition 2 hold. In equilibrium, when $alpha_t$ is larger than $\alpha^{\dagger}$, (at most) two consecutive screening offers must be followed by a waiting offer.*

The idea is that, when the seller can frequently revise the price, he always prefers to speed up the screening of the long-lived buyer. Hence, after being rejected once or twice, the seller believes that the long-lived buyer's value is too low so that he starts to post waiting offers. After several periods, the seller's belief about the existence of arrivals shifts downward, but his belief about the long-lived buyer's value remains; so he starts to screen the long-lived buyer again, and the screening process is very fast again. As a result, the seller frequently switches between waiting offers and screening offers in equilibrium.

6 Concluding Remarks

In this paper, we study a dynamic bargaining game between a long-lived seller and a long-lived buyer. The seller makes all offers, and the long-lived buyer has private information about his value. There may exist a short-lived player whose value is commonly known to be high. Conditional on his existence, the short-lived buyer's arrival is determined by a Poisson process. We characterize the unique equilibrium, which exhibits strategic delays and price fluctuations. In particular, when the seller is optimistic about the existence of the short-lived buyer, he charges a waiting offer, which is never accepted by the long-lived buyer. By making the waiting offer, the seller only adjusts her belief about the existence of short-lived buyers. Therefore, a waiting offer not only exploits the value of a potential new arrival but also controls the speed of learning. When the seller becomes sufficiently pessimistic about the existence of the short-lived buyer, she offers a price that is acceptable to the long-lived buyer with some values. If such offers are not accepted, the seller's beliefs about both the existence of new arrivals and the long-lived buyer's value change. The interaction between these two learning processes is the driving force of the price fluctuation.

We restrict our study to the situation where the short-lived buyer's value is commonly known and equals 1. One can easily extend the result to any $v_s < 1$. In addition, one can assume that there may be a sequence of short-lived buyers whose value are i.i.d. and who share the same distribution with the long-lived buyer's value. An arrival is observed only when the short-lived buyer takes the offer. In a model without the long-lived buyer, Mason and Valimaki (2011) show that the equilibrium waiting price declines over time. With the long-lived buyer, our conjecture is that the equilibrium price sequence frequently switches between two price paths: a screening price path and a waiting price path. There is another interesting extension of the model. Assume that there may be a sequence of short-lived buyers whose values are independent. Each short-lived buyer's value is either high or low. Only the seller can observe the arrival of the short-lived buyer. This is a difficult problem, because the seller and the long-lived buyer may have different beliefs on the equilibrium path. We leave this question for future research.

A Appendix

A.1 Equilibrium Construction

Proof of Lemma 1:

If v-buyer accepts p_τ in period τ, then $v - p_\tau \geq e^{-r\Delta}(1-\lambda\alpha\Delta)U(v,\alpha_\tau,H_\tau)$. We want to show that $v' - p_\tau > e^{-r\Delta}(1-\lambda\alpha\Delta)U(v',\alpha_\tau,H_\tau)$. Since from $\tau+1$ on, the v player can always adopt the optimal strategy of v'-buyer, that is, accept exactly when v'-buyer accepts, then

$$\begin{aligned} U(v,\alpha_\tau,H_\tau) &\geq \sum_{j=0}^{\infty} e^{-jr\Delta}\prod_{i=0}^{j}(1-\lambda\alpha_{\tau+i}\Delta)\gamma_{\tau+1+j}(v',\alpha,H_{\tau+1+j})(v-p_{\tau+j+1}), \\ U(v',\alpha_\tau,H_\tau) &= \sum_{j=0}^{\infty} e^{-jr\Delta}\prod_{i=0}^{j}(1-\lambda\alpha_{\tau+i}\Delta)\gamma_{\tau+1+j}(v',\alpha,H_{\tau+1+j})(v'-p_{\tau+j+1}), \end{aligned}$$

where $\gamma(v',\alpha_\tau,H_\tau)$ is the probability conditional on H_τ and α_τ that agreement is reached at time $\tau+1+j$ and the buyer use v'-buyer's optimal strategy from time $\tau+1$ on. In other words,

$$\begin{aligned} & U(v',\alpha_\tau,H_\tau) - U(v,\alpha_\tau,H_\tau) \\ \leq & \sum_{j=0}^{\infty} e^{-jr\Delta}\prod_{i=0}^{j}(1-\lambda\alpha_{\tau+j}\Delta)\gamma_{\tau+1+j}(v',H_\tau)(v'-v) \\ \leq & \; v'-v. \end{aligned} \tag{7}$$

Since $v - p_\tau \geq e^{-r\Delta}(1-\lambda\alpha_\tau\Delta)U(v,\alpha_\tau,H_\tau)$, we have

$$\begin{aligned} & v' - p_\tau - e^{-r\Delta}U(v',\alpha_\tau,H_\tau) \\ \geq & \; v' - (v - e^{-r\Delta}U(v,\alpha_\tau,H_\tau)) - e^{-r\Delta}U(v',\alpha_\tau,H_\tau) \\ = & \; (v'-v) - e^{-r\Delta}[U(v',\alpha_\tau,H_\tau) - U(v,\alpha_\tau,H_\tau)] \\ = & \; (v'-v) - [U(v',\alpha_\tau,H_\tau) - U(v,\alpha_\tau,H_\tau)] \\ & \qquad + (1-e^{-r\Delta})[U(v',\alpha_\tau,H_\tau) - U(v,\alpha_\tau,H_\tau)]. \end{aligned}$$

By (7),

$$(v'-v) - [U(v',\alpha_\tau,H_\tau) - U(v,\alpha_\tau,H_\tau)] \geq 0,$$

and

$$(1-e^{-r\Delta})[U(v',\alpha_\tau,H_\tau) - U(v,\alpha_\tau,H_\tau)] > 0,$$

we have $v' - p_\tau > e^{-r\Delta}(1-\lambda)U(v',\alpha_\tau,H_\tau)$. *Q.E.D.*

Proof of Proposition 1:

We first show the screening ends in finitely many periods in any equilibrium. Then, we construct the unique equilibrium by induction. The following lemma shows that screening offers are suboptimal, when the seller is sufficiently pessimistic about the long-lived buyer's value.

Lemma 3. *There is a $K^* > \underline{v}$ such that screening is not optimal when $k \in [\underline{v}, K^*)$.*

Proof. The seller can switch to the strategy "waiting offer forever" and obtain the expected value J_0 in any period. We first show that once the seller optimally posts one period waiting offer, he would not use screening offers in future. Suppose the seller posts a waiting offer in period t, it must be true that

$$R(k) = J(k) > V(k).$$

The game goes to period $t+1$ if no short-lived buyers appears. By posting the waiting offer, the seller cannot screen the long-lived buyer's value. Thus, $k' = k$, and $V(k') = V(k) < J(k) = J(k')$. Hence, the seller will post the waiting offer in period $t+1$. As a result, once the seller posts a waiting offer in period t, he will post the waiting offer until the game is over. Hence, $J(k) = J_0$, when $J(k) > V(k)$.

Now we want to show that there is a $K^* \in (\underline{v}, 1]$ such that the seller will switch to the waiting offer when $k < K^*$. Suppose not, then for any $k \in [\underline{v}, 1]$, $J(k) \leq V(k)$. However, $V(k) \leq k$, so when k is sufficiently close to $\underline{v}$, $V(k)$ is strictly less than J_0.

There are two cases. First, $J_0 > V(1)$, then $K^* = 1$. The seller will posts waiting offer from period 1 on. In this case, the equilibrium is trivial. In the second case, $J_0 \leq V(1)$. By monotonicity of $V(k)$, there exists a K^* such that $V(k) \geq J_0$ if and only if $k \in [K^*, 1]$. Given the existence of K^*, the seller will finally switch to the waiting offer in any equilibrium, if it exists. □

Now we show the existence of the equilibrium and its uniqueness by construction. Because the strategy "waiting offer forever" provides a lower bound of the seller's value, which is strictly higher than $\underline{v}$, this proof is essentially same as that in Ausubel and Deneckere (1989). However, the existence of the short-lived buyer makes the proof more complicated.

Step 1. We set

$$W^0(k) = J_0 \quad \text{and} \quad \sigma^0(k) = 1,$$

where $W^0(k)$ is the seller's payoff from charging the waiting offer 1, and $\sigma^0(k)$ is seller's optimal choice correspondence, which is single valued in this zero step screening game by construction. Denote by $\beta^1(k)$ the price that k-value long-lived buyer is indifferent between taking and having no trade. Then $\beta^1(k)$ is such that

$$k - \beta^1(k) = 0.$$

That is, suppose the seller's strategy is is to make a screening offer in the current period and begin to charge the waiting offer forever if the current offer is rejected, the k-value long-lived buyer is indifferent between accepting $\beta^1(k)$ and having no trade. Accordingly, if this last screening offer p is rejected, the upper bound of the seller's belief about the long-lived buyer's value is $\kappa^1(p) = p$. Obviously, $(\beta^1)^{-1}(p) = \kappa^1(p)$. By construction, $\beta^1(k)$ and $\kappa^1(p)$ are continuous and increasing.

Denote by $W^1(k)$ the seller's value from charging one screening offer and then switching to the waiting offer forever, so

$$W^1(k) = \max_{k' \leq k} [\frac{F(k) - F(k')}{F(k)} + \frac{F(k')}{F(k)}\lambda\Delta]p + \frac{F(k')}{F(k)}(1 - \lambda\Delta)e^{-r\Delta}J_0.$$

Let $T^1(k)$ be the maximum of the set of solutions to this optimization problem, and $T^1(k)$ exists by the maximum theorem in Ausubel and Deneckere (1993). Then we can define

$\sigma^1(k) = \beta^1(T^1(k))$ to be the seller's price at state variable k in this constructed game. Define $k^0 = \max\{k \in [\underline{v}, 1] | W^1(k) = W^0(k) = J_0\}$. By definition, because of the existence of K^*, k^0 is well defined. By this definition, for any $k \leq k^0$, the seller will switch to the waiting offer, and the long-lived buyer will reject such an offer, no matter what his value is. With $(\sigma^1, \kappa^1; W^1, \beta^1, T^1, k^0)$, we can begin our equilibrium construction.

Step 2. Set $k^{n+1} > k^n$ such that

$$F\left(k^{n+1}\right) = \min\{1, F\left(k^n\right)(1 - \lambda\Delta) + (\lambda\Delta + r\Delta)W^n\left(k^n\right)F\left(k^n\right)\}$$

thus when $k^{n+1} < 1$,

$$F\left(k^{n+1}\right) - F\left(k^n\right)(1 - \lambda\Delta) = F\left(k^n\right)(\lambda\Delta + r\Delta)W^n\left(k^n\right).$$

Let $\beta^{n+1}(k)$ be the price will makes the k-value long-lived buyer indifferent between taking the current offer and waiting for the next period offer. Then,

$$k - \beta^{n+1}(k) = (1 - \lambda\Delta - r\Delta)(k - \sigma^n(k)).$$

Here, $\beta^{n+1}(k)$ is non-decreasing in k. With this definition, we can define $W^{n+1}(k)$ be the value of the following optimization problem:

$$W^{n+1}\left(k\right) = \max_{k' \leq k} \frac{F\left(k\right) - F\left(k'\right)(1 - \lambda\Delta)}{F\left(k\right)}\beta^{n+1}\left(k'\right) + \frac{F\left(k'\right)(1 - \lambda\Delta)}{F\left(k\right)}e^{-r\Delta}W^n\left(k'\right),$$

Denote by $T^{n+1}(k)$ the supremum of the set of solutions to this optimization problem for $k \in [\underline{v}, k^{n+1}]$. We now claim that $T^{n+1} \leq k^n$.

Suppose not, then there is a $k \in [\underline{v}, k^{n+1}]$ such that $v = T^{n+1}(k)$ and $v \in (k^n, k^{n+1}]$. By approximating $e^{-r\Delta}$ by $1 - r\Delta$ and ignoring $O(\Delta^2)$ terms, we have

$$\begin{aligned}
& \frac{F\left(k\right) - F\left(v\right)(1 - \lambda\Delta)}{F\left(k\right)}\beta^{n+1}\left(v\right) + (1 - \lambda\Delta - r\Delta)\frac{F\left(v\right)}{F\left(k\right)}W^{n+1}\left(v\right) \\
< \; & \frac{F\left(k^{n+1}\right) - F\left(k^n\right)(1 - \lambda\Delta)}{F\left(k\right)} + (1 - \lambda\Delta - r\Delta)W^{n+1}\left(k\right) \\
\leq \; & (\lambda\Delta + r\Delta)W^n\left(k^n\right) + (1 - \lambda\Delta - r\Delta)W^{n+1}\left(k\right) \\
\leq \; & (\lambda\Delta + r\Delta)W^{n+1}\left(k\right) + (1 - \lambda\Delta - r\Delta)W^{n+1}\left(k\right) \\
= \; & W^{n+1}\left(k\right).
\end{aligned}$$

This leads to a contradiction! Let $\sigma^{n+1}(k) = \beta^{n+1}[T^{n+1}(k)]$. $\beta^{n+1}(k)$ may have finitely many jumps, and it is right continuous. Define $\kappa^{n+1}(p)$ as follows: (1) $\kappa^{n+1}(p) = (\beta^{n+1})^{-1}(p)$ when $\beta^{n+1}(k)$ is invertible, or (2) if there is $\hat{k}$ such that $\lim_{k \searrow \hat{k}} \beta^{n+1}(k) = p^+ \neq \lim_{k \nearrow \hat{k}} \beta^{n+1}(k) = p^-$, $\kappa^{n+1}(p) = \hat{k}$ for $p \in (p^-, p^+]$. Therefore, by construction, $\kappa^{n+1}\left(p\right)$ is a continuous function.

Step 3. The construction of κ^{n+1} in **Step 2** suggests to us that there may be more than one price, which induces the same state variable k' in the next period. On the equilibrium path, this does not cause any problem, because there is a unique sequence of prices, and for each price p in this sequence, we have assigned an appropriate k, and given k the continuation play is described by (σ^n, κ^n), where σ^n does not depend on p. However, to complete a strategy

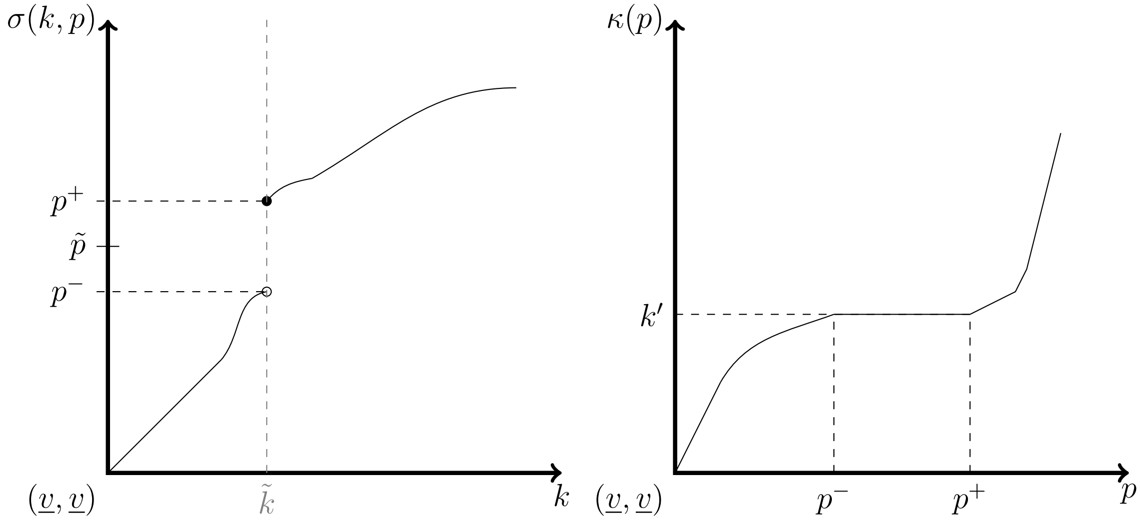

Figure 2: Pricing function σ and cutoff function κ.

profile, we also have to describe the continuation play after any deviation of the seller. In this case, the flat part of the constructed κ^{n+1} causes σ^n to depend on the deviating price. Figure 2 illustrates this case.

When we construct κ^{n+1}, if there is $\hat{k}$ such that

$$\lim_{k \searrow \hat{k}} \beta^{n+1}(k) = p^+ \neq \lim_{k \nearrow \hat{k}} \beta^{n+1}(k) = p^-,$$

$\kappa^{n+1}(p) = \hat{k}$ for $p \in (p^-, p^+]$. This κ is illustrated in part (b) of Figure 2. When the seller deviates to a price $\hat{p} \in (p^-, p^+)$, the equilibrium construction assigns $\hat{k}$ as the next period upper bound of the seller's belief about the long-lived buyer's value. For this belief to be correct, the $\hat{k}$-value long-lived buyer must be indifferent between taking the current offer $\hat{p}$ and waiting for future offers. However, if the seller chooses p'^+ in the next period given $\hat{k}$, the $\hat{k}$-value long-lived buyer will strictly prefer the current offer; and if the seller chooses p'^- in the next period given $\hat{k}$, the $\hat{k}$-value long-lived buyer will strictly prefer waiting. This implies that to make the $\hat{k}$-value long-lived buyer indifferent, the seller's price strategy in the next period given $\hat{k}$ must be mixed. For the seller to randomize given $\hat{k}$, the seller must be indifferent between the continuation strategies in the support of the mixed strategy in the next period. This requires that any continuation strategies in the support of the seller's mixed strategy in the next period given $\hat{k}$ must be part of an equilibrium at the state variable $\hat{k}$. Therefore, after a deviating price $\hat{p}$, the seller's strategy depends on the value of $\hat{p}$, in the way that the seller appropriately randomizes among equilibrium continuation strategies given $\hat{k}$. Hence, in an equilibrium, it is possible that on the off-equilibrium path, the price depends on the last period price. This reconciles the discussion in section **??** about weak-Markov equilibrium and strong-Markov equilibrium.

In **Step 2**, we specify that on the equilibrium path, to reach the next period state variable k', the seller charges the highest possible price. If there are other prices also leading to the same k', the seller's continuation value is the same, but the seller gets higher current payoff. If there is no other price leading to the same k', the construction itself shows that the seller's strategy is the best response to the long-lived buyer's strategy. Therefore, if (σ^n, κ^n) describes an equilibrium for all $k \in [\underline{v}, k^n]$, $(\sigma^{n+1}, \kappa^{n+1})$ describes an equilibrium for all $k \in [\underline{v}, k^{n+1}]$.

Step 4. To complete the argument that the constructed (σ, κ) describes an equilibrium, we need to show that there is $N \in \mathbb{N}$, such that $k^{N+1} \geq 1$. Recall that

$$F\left(k^{n+1}\right) - F\left(k^n\right) = F\left(k^n\right)[-\lambda\Delta + (\lambda\Delta + r\Delta)W^n\left(k^n\right)].$$

Because $W^n(k) \geq W^{n-1}(k) \geq ...W^1(k) > J_0 = \frac{\lambda\Delta}{r\Delta+\lambda\Delta}$ for any $k > k^0$, $F(k^{n+1}) - F(k^n) > 0$ and increases in n. Since the density function is bounded above by $\bar{f}$, there exists $N \in \mathbb{N}$, such that $k^{N+1} \geq 1$. *Q.E.D.*

Proof of Proposition 2:

By construction, the sequence of prices $\{p_t\}$ is decreasing for all $t \leq N$, and from period $N+1$ on, $p_t = 1$. So to show the lower bound of the equilibrium prices, we just need to show that $p_N \geq \frac{\lambda}{\lambda+\gamma} = J_0$. Suppose $p_N < \frac{\lambda}{\lambda+\gamma}$. Then if p_N is taken, the seller's payoff is $p_N < J_0$. If p_N is not taken, the seller will charge the waiting offer from the next period. In this case, her payoff is also strictly smaller than J_0. Therefore, in period N, the seller would like to begin to charge the waiting offer from period N on. This is contradicted to Proposition 1. Therefore, $p_t \geq \frac{\lambda}{\lambda+\gamma}$. The second part of the statement comes from Coasian conjecture in gap case. The proof is omitted since it is standard. *Q.E.D.*

Proof of Lemma 2:

Since we haven't compared $R(k,\alpha)$ and $R(k,\alpha')$, we discuss two cases. In the first case, assume $R(k,\alpha) \leq R(k,\alpha')$. By definition, $J(k,\alpha) = \lambda\alpha\Delta + (1-\lambda\alpha\Delta)(1-r\Delta)R(k,\alpha')$, which is no larger than $R(k,\alpha)$ and $R(k,\alpha')$ (and $R(k,\alpha) \geq \underline{v}$). The waiting offer is strictly suboptimal if

$$\lambda\alpha\Delta \leq \frac{R(k,\alpha) - e^{-r\Delta}R(k,\alpha')}{1 - e^{-r\Delta}R(k,\alpha')} \leq \frac{R(k,\alpha') - e^{-r\Delta}R(k,\alpha')}{1 - e^{-r\Delta}R(k,\alpha')} = \frac{1 - e^{-r\Delta}}{1/R(k,\alpha') - e^{-r\Delta}}.$$

The right hand side is increasing in $R(k,\alpha')$. Since $R(k,\alpha') \geq \underline{v}$, simple algebra implies that waiting offer will not be posted if $\alpha \leq \alpha^\dagger = \frac{1}{\lambda\Delta}\frac{1-e^{-r\Delta}}{1/\underline{v}-e^{-r\Delta}}$.

In the second case, assume $R(k,\alpha) > R(k,\alpha')$. A sufficient condition for

$$R(k,\alpha) = V(k,\alpha) > \lambda\alpha\Delta + (1 - \lambda\alpha\Delta - r\Delta)R(k,\alpha') \tag{8}$$

is

$$R(k,\alpha) \geq \lambda\alpha\Delta + (1 - \lambda\alpha\Delta - r\Delta)R(k,\alpha).$$

This holds if and only if $(\lambda\alpha + r)R(k,\alpha) \geq \lambda\alpha$, which is true if and only if $\frac{rR(k,\alpha)}{1-R(k,\alpha)} \geq \lambda\alpha$. Since $R(k,\alpha) \geq \underline{v}$, it is obvious that if $\alpha < \alpha^\dagger$, the equation (8) holds. *Q.E.D.*

Proof of Proposition 3:

We first show the existence of the equilibrium and its uniqueness of a continuation game starting from (k, α), where $\alpha \in B_\Delta(\alpha^\dagger)$.

Step 1. For each $\alpha \in B_\Delta\left(\alpha^\dagger\right)$, define for all (k,α) with $k \in [\underline{v}, 1]$ and $\alpha \in B_\Delta(\alpha^\dagger)$,

$$W^0(k,\alpha) = \underline{v}, \quad \sigma^0(k,\alpha) = \underline{v}, \quad \text{and} \quad \kappa^0(\underline{v},\alpha) = \underline{v},$$

where $W^0(k,\alpha)$ is the seller's value from posting the offer $\sigma^0(k,\alpha) = \underline{v}$, and the long-lived buyer with a value bigger than or equal to $\underline{v}$ accepts this offer. Now define $\beta^1(k,\alpha)$ and $\kappa^1(p,\alpha)$ such that

$$\begin{aligned} k - \beta^1(k,\alpha) &= (1 - \lambda\alpha\Delta - r\Delta)(k - \underline{v}), \\ \kappa^1(p,\alpha) - p &= (1 - \lambda\alpha\Delta - r\Delta)\left(\kappa^1(p,\alpha) - \underline{v}\right); \end{aligned}$$

thus, $\kappa^1(\beta^1(k,\alpha),\alpha) = k$. Moreover, define

$$\begin{aligned} & W^1(k,\alpha) \\ = & \max_{k' \le k} \frac{F(k) - F(k')(1 - \lambda\alpha\Delta)}{F(k)} \beta^1(k',\alpha) \\ & \qquad + \frac{F(k')(1-\lambda\alpha\Delta)}{F(k)} e^{-r\Delta} W^0(k',\alpha'). \end{aligned}$$

Let $T^1(k,\alpha)$ be the maximum of the set of solutions to this optimization problem, we can define $\sigma^1(k,\alpha) = \beta^1(T^1(k,\alpha),\alpha')$.

Define

$$k^0(\alpha) = \max\left\{k | W^1(k,\alpha) = \underline{v}\right\}.$$

So when $k < k^0(\alpha)$, the seller will charge price $\sigma(k,\alpha) = \underline{v}$. Obviously, in this constructed zero stage game, (σ^0, κ^0) describes an equilibrium for $k \in [\underline{v}, k^0(\alpha))$.

Step 2. With $(\sigma^1, \kappa^1; W^1, \beta^1, T^1, k^0)$, we now construct the equilibrium by induction. For each $k^n(\alpha')$, let $k^{n+1}(\alpha)$ be the largest k such that

$$F(k) = \min\{1, F(k^n(\alpha'))[1 + \frac{\lambda\alpha\Delta(1 - \beta^{n+1}(k,\alpha))}{\beta^{n+1}(k,\alpha) - \underline{v}}]\},$$

for all $n \ge 0$.

$$F(k^{n+1}(\alpha)) \le F(k^n(\alpha'))[1 + \frac{\lambda\alpha\Delta(1 - \beta^{n+1}(k,\alpha))}{\beta^{n+1}(k,\alpha) - \underline{v}}].$$

For each $\alpha \in B_\Delta\left(\alpha^\dagger\right)$, if the seller chooses a cutoff value k as the next period state variable, she need to make the k-value long-lived buyer indifferent between the current offer and the offer in the next period. Since (k,α') will be the next period state variable, in the next period offer will be, by definition, $\sigma^n(k,\alpha')$. Therefore, the current offer, which makes the k-value long-lived buyer indifferent would be $\beta^{n+1}(k,\alpha)$ such that

$$k - \beta^{n+1}(k,\alpha) = (1 - \lambda\alpha\Delta)e^{-r\Delta}(k - \sigma^n(k,\alpha')).$$

Using this definition, we can define $W^{n+1}(k,\alpha)$ as the value of the following optimization problem:

$$\begin{aligned} & W^{n+1}(k,\alpha) \\ = & \max_{k' \le k} \frac{F(k) - F(k')(1 - \lambda\alpha\Delta)}{F(k)} \beta^{n+1}(k',\alpha) \\ & \qquad + \frac{F(k')(1-\lambda\alpha\Delta)}{F(k)} e^{-r\Delta} W^n(k',\alpha'). \end{aligned} \tag{9}$$

We claim that for any $k \in [k^n(\alpha'^{n+1}(\alpha))$, the maximum of the set of solutions to the optimization problem 9 $T^{n+1}(k,\alpha) < k^n(\alpha')$.

Suppose not, then let $v = T^{n+1}(k,\alpha) \geq k^n(\alpha')$. Consider the following two conditions. First, because $\alpha \leq \alpha^{\dagger}$,

$$W^{n+1}(v,\alpha) > \lambda\alpha\Delta + (1-\lambda\alpha\Delta)W^{n+1}(v,\alpha').$$

Second, since the seller at the state variable (k,α) can employ the strategy using at (v,α), we have

$$F(k)W^{n+1}(k,\alpha) \geq F(v)W^{n+1}(v,\alpha) + p(F(k)-F(v)),$$

where p is the price making the long-lived buyer with value $T^{n+1}(v,\alpha)$ indifferent between taking the offer p and waiting for another period. These two conditions imply

$$\begin{aligned} & F(k)W^{n+1}(k,\alpha) \\ > \; & \lambda\alpha\Delta F(v) + (1-\lambda\alpha\Delta - r\Delta)F(v)W^{n+1}(v,\alpha') + p(F(k)-F(v)), \end{aligned}$$

which in turn implies that

$$\begin{aligned} & (1-\lambda\alpha\Delta - r\Delta)F(v)W^{n+1}(v,\alpha') \\ < \; & F(k)W^{n+1}(k,\alpha) - \lambda\alpha\Delta F(v) - p(F(k)-F(v)). \end{aligned}$$

Now, we have

$$\begin{aligned} & F(k)W^{n+1}(k,\alpha) \\ = \; & [F(k)-F(v)(1-\lambda\alpha\Delta)]\beta^{n+1}(v,\alpha) + F(v)(1-\lambda\alpha\Delta - r\Delta)\underline{v} \\ \leq \; & [F(k)-F(v)(1-\lambda\alpha\Delta)]\beta^{n+1}(v,\alpha) \\ & \qquad + F(v)(1-\lambda\alpha\Delta - r\Delta)W^{n+1}(v,\alpha') \\ < \; & [F(k)-F(v)(1-\lambda\alpha\Delta)]\beta^{n+1}(v,\alpha) \\ & \qquad + F(k)W^{n+1}(k,\alpha) - \lambda\alpha\Delta F(v) - p(F(k)-F(v)) \\ = \; & [F(k)-F(v)](\beta^{n+1}(v,\alpha)-p) + F(k)W^{n+1}(k,\alpha) - \lambda\alpha\Delta F(v)[1-\beta^{n+1}(v,\alpha)] \\ \leq \; & [F(k^{n+1}(\alpha)) - F(k^n(\alpha'^{n+1}(v,\alpha) - \underline{v}) \\ & \qquad + F(k)W^{n+1}(k,\alpha) - \lambda\alpha\Delta F(k^n(\alpha'^{n+1}(v,\alpha)] \\ = \; & F(k)W^{n+1}(k,\alpha). \end{aligned}$$

This leads to the contradiction.

Step 3. Similar to the equilibrium construction in the no learning case, we have to specify the continuation play after the seller's deviation. For any k', which is induced by a unique price p in the current period, the continuation play is as constructed in **Step 2**. So the construction in **Step 2** shows that deviating to such a price is not profitable. If multiple prices induce a same k', the seller must randomize among continuation strategies, each of which is part of an equilibrium of the game beginning at (k',α'). In addition, how the seller randomizes depends on the deviating price, that is, the seller will choose an appropriate mixed strategy to make the k'-value long-lived buyer indifferent between taking the current deviating price and waiting for the next period offer. On the equilibrium path, the seller charges the highest price inducing k', so after the deviation, the seller's continuation payoff is the same as that on the equilibrium path. But the seller has higher current payoff by following the equilibrium price, so such a deviation is not profitable either. Therefore, if (σ^n, κ^n) describes an equilibrium for all $\alpha \in B_\Delta(\alpha^\dagger)$ and all $k \in [\underline{v}, k^n(\alpha))$, $(\sigma^{n+1}, \kappa^{n+1})$ describes an equilibrium for all $\alpha \in B_\Delta(\alpha^\dagger)$ and all $k \in [\underline{v}, k^{n+1}(\alpha))$.

Step 4. Now we show that a finite number of repetitions of the above argument extends $\left(\sigma^{N^*_\alpha}, \kappa^{N^*_\alpha}; W^{N^*_\alpha}, \beta^{N^*_\alpha}, T^{N^*_\alpha}, k^{N^*_\alpha - 1}\right)$ to $k \in [\underline{v}, 1]$ for any $\alpha \in B_\Delta\left(\alpha^\dagger\right)$. By the construction of sequences of $\{k^n(\alpha)\}$ for all $\alpha \in B_\Delta(\alpha^\dagger)$, suppose for any finite number N_α, $k^{N_\alpha}(\alpha) < 1$. There are two cases. In the first case, there is an $\epsilon > 0$, such that $k^n(\alpha) < 1 - \epsilon$. But then

$$\begin{aligned} & F(k^{n+1}(\alpha)) - F(k^n(\alpha')) \\ = \; & F(k^n(\alpha))\lambda\alpha\Delta \frac{1 - \beta^{n+1}(k^{n+1}(\alpha), \alpha)}{\beta^{n+1}(k^{n+1}(\alpha), \alpha) - \underline{v}} \\ \geq \; & F(k^n(\alpha))\lambda\alpha\Delta \frac{\epsilon}{1 - \epsilon - \underline{v}}. \end{aligned}$$

Because the density function is bounded above by $\bar{f}$, we can find the contradiction. In the second case, for any $\epsilon > 0$, there is N such that for all $n \geq N$, $k^n(\alpha) > 1 - \epsilon$. Since

$$\begin{aligned} & F(k^{n+1}(\alpha)) - F(k^n(\alpha')) \\ = \; & F(k^n(\alpha))\lambda\alpha\Delta \frac{1 - \beta^{n+1}(k^{n+1}(\alpha), \alpha)}{\beta^{n+1}(k^{n+1}(\alpha), \alpha) - \underline{v}}, \end{aligned}$$

$f'(1) = 0$ implies $k^{n+1}(\alpha) - k^n(\alpha')$ is bounded away from 0. So we also get the contradiction. *Q.E.D.*

Proof of Proposition 4:

By Lemma 2, the waiting offer will not be posted when $\alpha_t \in B_\Delta(\alpha^\dagger)$, and any continuation game, which starts with (k, α) such that $k \in [\underline{v}, 1]$, $\alpha \in B_\Delta(\alpha^\dagger)$, ends in $N^*_{\alpha^\dagger}$ periods. Hence, the seller's continuation value and associated pricing policy, $R(k, \alpha^\dagger)$ and $\sigma(k, \alpha^\dagger)$ can be calculated for all $k \in [\underline{v}, 1]$. And the k-buyer's equilibrium payoff $U\left(k, k, \alpha^\dagger | \sigma, \kappa\right)$ under the continuation play can also be calculated.

We move to (k, α^2) where $k \in [\underline{v}, 1]$. First, any $(k, \alpha^\dagger)$ can be reached from (k, α^2) by one period waiting offer. By making a waiting offer, the seller's payoff is given by

$$J\left(k, \alpha^2\right) = \lambda\alpha^2\Delta + \left(1 - \lambda\alpha^2\Delta - r\Delta\right) R\left(k, \alpha^\dagger\right),$$

the buyer has no relevant choice to make, and the game ends in $N^*_{\alpha^\dagger} + 1$ periods.

Second, construct $\beta\left(k, \alpha^2\right)$ as follows.

$$k - \beta\left(k, \alpha^2\right) = \left(1 - \lambda\alpha^2 - r\Delta\right) U\left(k, k, \alpha^\dagger | \sigma, \kappa\right),$$

where $\beta\left(k, \alpha^2\right)$ is the highest price such that k-buyer is indifferent between taking it and waiting to obtain a continuation payoff $U\left(k, k, \alpha^\dagger | \sigma, \kappa\right)$. Following Ausubel and Deneckere (1989), without loss of generality, assume $\sigma^s\left(k, \alpha^2\right)$ is monotone and right continuous. Define $\kappa\left(\beta\left(k, \alpha^2\right), \alpha^2\right) = k$ if it is well defined. As Ausubel and Deneckere (1989) noted, $\beta\left(k, \alpha^2\right)$ may have jumps at finitely many points. For any jump point $\hat{k}$ such that $\beta\left(\hat{k}, \alpha^2\right) = \hat{p}_+$

and $\lim_{k\to\hat{k}^-}\beta(k,\alpha^2)=\hat{p}_-$, where $\hat{p}_-<\hat{p}_+$, let $\kappa(p,\alpha^2)=\hat{k}$ for all $p\in[\hat{p}_-,\hat{p}_+]$. Hence, by construction, $\kappa(p,\alpha^2)$ is continuous with respect to p, and $\kappa^{-1}(k|\alpha^2)=\beta(k,\alpha^2)$ is defined.

For any $k\in[\underline{v},1]$, we define the seller's value by posting an optimal screening offer as follows:

$$V(k,\alpha^2)=\max_{k'\in k}[1-\frac{F(k')}{F(k)}(1-\lambda\alpha^2\Delta)]\beta\left(k',\alpha^2\right)+\frac{F(k')}{F(k)}(1-\lambda\alpha^2\Delta)e^{-r\Delta}R(k',\alpha^\dagger).$$

Let $T^s(k,\alpha^2)$ be the maximum of the set of solutions to this optimization problem, and $\sigma^s(k,\alpha^2)=\beta(T^s(k,\alpha^2),\alpha^2)$. Since both $R(k,\alpha^\dagger)$ and $\beta(k,\alpha^2)$ is well defined, $V(k,\alpha^2)$ and $T^s(k,\alpha^2)$ are well defined. By posting an optimal screening offer $\sigma^s(k,\alpha^2)$ in the current period, the seller obtains $V(k,\alpha^2)$, and the game ends in at most $N^*_{\alpha^\dagger}+1$ periods.

Define $R(k,\alpha^2)=\max\{V(k,\alpha^2),J(k,\alpha^2)\}$ as the seller's value by choosing an optimal offer with a price

$$\sigma(k,\alpha)=\begin{cases}\sigma^s(k,\alpha), & \text{if} \quad V(k,\alpha)\geq J(k,\alpha)\\ 1, & \text{otherwise}\end{cases}$$

Hence, we extend the equilibrium to (k,α^2) for any $k\in[\underline{v},1]$. The game ends in at most $N^*_{\alpha^\dagger}+1$ periods regardless of the choice of offers in the current period. Given the continuation payoff at (k,α^2) for any $k\in[\underline{v},1]$, the k-buyer's continuation payoff, $U(k,k,\alpha^2|\sigma,\kappa)$ can be calculated.

Since, from α_0 to $\alpha_t=\alpha^\dagger$, there are finitely many periods, we can apply the above arguments for finitely many times to extend the equilibrium to (k,α_0) for any $k\in[\underline{v},1]$. *Q.E.D.*

A.2 Equilibrium Properties

Proof of Proposition 5:

The value of outside option by waiting forever is given by

$$\begin{aligned}J_0(\alpha_t) &= \lambda\alpha_t\Delta+(1-\lambda\alpha_t\Delta)e^{-r\Delta}\lambda\alpha_{t+\Delta}\Delta\\ &\quad+(1-\lambda\alpha_t\Delta)(1-\lambda\alpha_{t+\Delta}\Delta)e^{-2r\Delta}\lambda\alpha_{t+2\Delta}\Delta+....\end{aligned}$$

Since α_t is strictly decreasing, for any α_t, we have

$$\begin{aligned}J_0(\alpha_t) &< \lambda\alpha_t\Delta+(1-\lambda\alpha_t\Delta)e^{-r\Delta}\lambda\alpha_t\Delta\\ &\quad+(1-\lambda\alpha_t\Delta)(1-\lambda\alpha_t\Delta)e^{-2r\Delta}\lambda\alpha_t\Delta+....\end{aligned}$$

or

$$J_0(\alpha_t)<\frac{\lambda\alpha_t\Delta}{1-e^{-r\Delta}(1-\lambda\alpha_t\Delta)}.$$

Also, $J_0(\alpha_t)$ satisfies the following recursive equation:

$$J_0(\alpha_t)=\lambda\alpha_t\Delta+(1-\lambda\alpha_t\Delta)e^{-r\Delta}J_0(\alpha_{t+\Delta}).$$

Hence, we have

$$\begin{aligned}J_0(\alpha_t)-J_0(\alpha_{t+\Delta}) &= \lambda\alpha_t\Delta+\left[(1-\lambda\alpha_t\Delta)e^{-r\Delta}-1\right]J_0(\alpha_{t+\Delta})\\ &> \lambda\alpha_t\Delta-\lambda\alpha_{t+\Delta}\Delta>0\end{aligned}$$

and therefore J_0 is strictly increasing in α_t. For small α, $J_0(\alpha) < \underline{v}$, but there is a $\underline{\alpha}(\Delta)$ such that $J_0(\alpha) > \underline{v}$ for all $\alpha > \underline{\alpha}(\Delta)$. When $J_0(\alpha)$ is greater than $\underline{v}$, the seller does not want to post $p = \underline{v}$. Hence, fix Δ and T, there is a sufficiently large $\alpha_0 < 1$ such that $\alpha_T > \underline{\alpha}(\Delta)$. Thus, $p_t > \underline{v}$. Since the updating process of α_t is exogenous, for any finite T, we can find an α_0 large enough but smaller than one such that $\alpha_T > \underline{\alpha}(\Delta)$. Take the smallest α_0 such that $\alpha_T > \underline{\alpha}(\Delta)$ as $\bar{\alpha}$.

Q.E.D.

Proof of Proposition 6:

Suppose not. Consider the following equilibrium outcome induced by an equilibrium strategy profile which can be described by (σ, κ), the state evolves as follows: on the path of play, suppose that a screening offer p_{-1} induces a state (k, α). The seller optimally posts two consecutive screening offers p_1 and p_2 in the following two periods, which induce state variables (k', α') and (k'', α'') respectively. We will construct a profitable deviation such that (1) the seller posts screening offer $\tilde{p}$ instead of p_1, and induces (k'', α').

By using three consecutive screening offers, the seller's payoff can be decomposed into three parts: the flow payoff in two periods, and the continuation payoff conditional on no trade.

The flow payoff in the first period is

$$[(\frac{F(k) - F(k')}{F(k)}) + \frac{F(k')}{F(k)}\lambda\Delta\alpha]p_1,$$

where $(\frac{F(k)-F(k')}{F(k)})$ is the probability that the long-lived buyer takes the offer, and $\frac{F(k')}{F(k)}\lambda\Delta\alpha$ is the probability that a short-lived buyer takes the offer in the first period.

The flow payoff in the second period is

$$\frac{F(k')}{F(k)}(1 - \lambda\Delta\alpha) \times (1 - r\Delta)\,[\frac{F(k') - F(k'')}{F(k')} + \frac{F(k'')}{F(k')}\lambda\Delta\alpha'']p_2,$$

where $\frac{F(k')}{F(k)}(1-\lambda\Delta\alpha_t)$ is the probability that no trade happens in the first period, $\frac{F(k')-F(k'')}{F(k')}$ is the conditional probability that the long-lived buyer takes the offer p_2, and $\frac{F(k'')}{F(k')}\lambda\Delta\alpha'$ is the conditional probability that the short-lived buyer takes the offer in the second period.

The continuation value is

$$\frac{F(k')}{F(k)}\frac{F(k^{''})}{F(k')}(1 - \lambda\Delta\alpha)(1 - \lambda\Delta\alpha')\,(1 - r\Delta)^2\,R(k'', \alpha''),$$

where $\frac{F(k')}{F(k)}\frac{F(k^{''})}{F(k')}(1 - \lambda\Delta\alpha)(1 - \lambda\Delta\alpha')$ is the probability that no trade happens in the first two periods, and $R(k'', \alpha'')$ is the seller's continuation value conditional on his belief on both long-lived buyer's value and short-lived buyer's existence.

Consider an alternative pricing strategy which we described above. By construction, κ is right continuous in p for any α; thus, the seller can always find the price $\tilde{p}$ such that k'-long-lived buyer is indifferent between taking $\tilde{p}$ given α_t and waiting.

The seller's payoff can be decomposed into two parts again: the flow payoff in the first period and the continuation payoff.

The flow payoff in the first period is

$$[(\frac{F(k)-F(k'')}{F(k)})+\frac{F(k'')}{F(k)}\Delta\lambda\alpha]\tilde{p},$$

where $\tilde{p}$ is the deviation price which can induce k''-long-lived buyer to be the marginal type.

Discounted continuation value is

$$\frac{F(k'')}{F(k)}(1-\lambda\Delta\alpha)\,(1-r\Delta)\,R\,(k'',\alpha')\,.$$

By definition, $R\,(k'',\alpha')\geq J\,(k'',\alpha')$; thus the lower bound of deviation payoff is

$$\tilde{R}\,(k,\alpha)=[(\frac{F(k)-F(k'')}{F(k)})+\frac{F(k'')}{F(k)}\Delta\lambda\alpha]\tilde{p}+\frac{F(k'')}{F(k)}(1-\lambda\Delta\alpha)\,(1-r\Delta)\,J\,(k'',\alpha')\,.$$

We are going to show that this lower bound is greater than the equilibrium payoff $R\,(k,\alpha)$ when Δ is small. Note that $J\,(k'',\alpha')$ can be decomposed into two parts: the flow payoff in the second period,

$$\frac{F(k'')}{F(k)}(1-\lambda\Delta\alpha)\,(1-r\Delta)\,\lambda\Delta\alpha'\times 1,$$

where the price is 1, and the continuation value,

$$\frac{F(k'')}{F(k)}(1-\lambda\Delta\alpha)(1-\lambda\Delta\alpha')\,(1-r\Delta)^2\,R(k'',\alpha'').$$

Multiplying $F(k)$ to the lower bound of the difference of the payoff induced by two strategy yields

$$\begin{aligned}&(F(k)-F(k'))(\tilde{p}-p_1)+(F(k')-F(k''))(\tilde{p}-(1-r\Delta)\,p_2)\\&+\Delta\lambda\alpha(F(k'')\tilde{p}-F(k')p_1)+F(k'')(1-\lambda\alpha)\,(1-r\Delta)\,\lambda\Delta\alpha'(1-p_2).\end{aligned}$$

The sum of first three terms of the difference is

$$\begin{aligned}&(F(k)-F(k'))(\tilde{p}-p_1)+(F(k')-F(k''))(\tilde{p}-(1-r\Delta)\,p_2)\\&+\lambda\alpha\Delta(F(k'')\tilde{p}-F(k')p_1).\end{aligned}$$

We claim it is $O(\Delta^2)$ because the following reason. In each period, there is a marginal type of the long-lived buyer who is indifferent between taking the offer now and waiting for a lower price latter. Hence, we have

$$k'-p_1=(1-r\Delta)\,(1-\lambda\alpha_t\Delta)\,(k'-p_2)\,,$$

in equilibrium; thus p_1-p_2 is $O(\Delta)$ from simple algebra. Same logic, $p_{-1}-p_1$ is also $O\,(\Delta)$. By condition 1, both $k-k'$ and $k'-k''$ are also at most $O(\Delta)$; thus, $F(k)-F(k^{'})\leq(k-k')\bar{f}$ and $F(k')-F(k^{''})\leq(k'-k'')\bar{f}$ are also at most $O(\Delta)$.

To induce k'' rather than k' in the first period, the seller has to decrease the price to $\tilde{p}<p_1$. By condition 2, we have that the difference between p_2 and $\tilde{p}$ is $O(\Delta)$. Hence, both $\tilde{p}-p_1$ and $\tilde{p}-(1-r\Delta)\,p_2$ are $O\,(\Delta)$.

The last term of the difference is

$$F(k'')(1-\lambda\Delta\alpha)\,(1-r\Delta)\,\lambda\Delta\alpha'(1-p_2)>0,$$

which is $O(\Delta)$ since k'' is bounded away from zero, and $p_2 < k' < 1$. Keep applying the above logic if $\sigma(k'',\alpha') \neq 1$. A finite number of repetitions of the above argument complete the proof.

Keep applying the above logic can prove that no more than N consecutive screening offer will show up in equilibrium when Δ is small. Note the cutoff Δ_α depends on α! Yet, on the path of play, there are finitely many realization of $\alpha \in B_\Delta(\alpha_0) \setminus B_\Delta(\alpha^\dagger)$, hence we can find the smallest $\bar{\Delta} = \min\{\Delta_\alpha\}$ for all possible $\alpha \in B_\Delta(\alpha_0) \setminus B_\Delta(\alpha^\dagger)$ such that when $\Delta \in (0, \bar{\Delta})$, the statement holds. *Q.E.D.*

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